

The Richard Stockton College of New Jersey  
Mathematical Mayhem 2014  
Group Round - Solutions

March 22, 2014

Name: \_\_\_\_\_

Name: \_\_\_\_\_

Name: \_\_\_\_\_

High School: \_\_\_\_\_

**Instructions:**

- This round consists of **5** problems worth **16** points each for a total of **80** points.
- Each of the 5 problems is free response.
- Write your complete solution in the space provided including all supporting work.
- No calculators are permitted.
- This round is **75 minutes** long. **Good Luck!**

---

**OFFICIAL USE ONLY:**

Problem #	1	2	3	4	5	Total
Points Earned						

• Group Round •

**Problem 1.** A point is selected at random inside an equilateral triangle. From this point perpendiculars are dropped to each side. Show the sum of these perpendiculars is equal to the altitude of the triangle.

**Solution to Question 1.** Let  $P$  be any point inside the equilateral  $\triangle ABC$  with sides length  $s$  and let the three perpendiculars be  $P_a, P_b, P_c$ .

$$\text{Area of } \triangle ABC = \text{Area of } \triangle APB + \text{Area of } \triangle BPC + \text{Area of } \triangle CPA$$

$$= \frac{1}{2}s(\phantom{P_a + P_b + P_c})$$

**Problem 3.** Start with a circle  $C$  with two smaller circles inscribed in it as the initial configuration, as shown in Figure (a) below. Consider the procedure *Add Circles* defined as follows.

*Add Circles:* Inscribe a circle into each non circular region.

The result of applying *Add Circles* to Figure (a) is Figure (b), shown below. Now repeat *Add Circles* four more times; in total, you have applied *Add Circles* to the initial circle configuration five times. After the fifth application of *Add Circles*, how many of the total circles pictured do not touch circle  $C$ ?

Suppose *Add Circles*

