The Richard Stockton College of New Jersey
Mathematical Mayhem 2013
Group Round

March 23, 2013

Name: ____

Name: ____

Name: ____

High School: ____

Instructions:

- This round consists of **5** problems worth **16** points each for a total of **80** points.
- Each of the 5 problems is free response.
- Write your complete solution in the space provided including all supporting work.
- No calculators are permitted.
- This round is **75 minutes** long. **Good Luck!**

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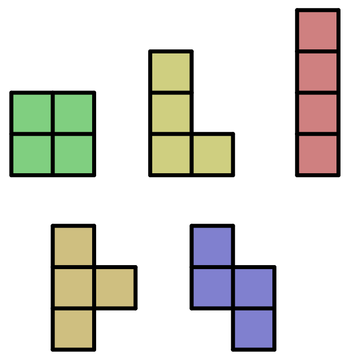
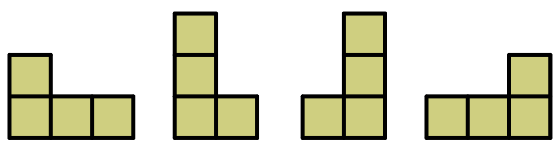
Problem #	1	2	3	4	5	Total
Points Earned						

• Group Round •

Problem 1. A polyomino is a contiguous shape formed by gluing together squares edge to edge. A polyomino made up of 4 squares is called a tetromino. There are 5 different tetrominoes, as shown below.

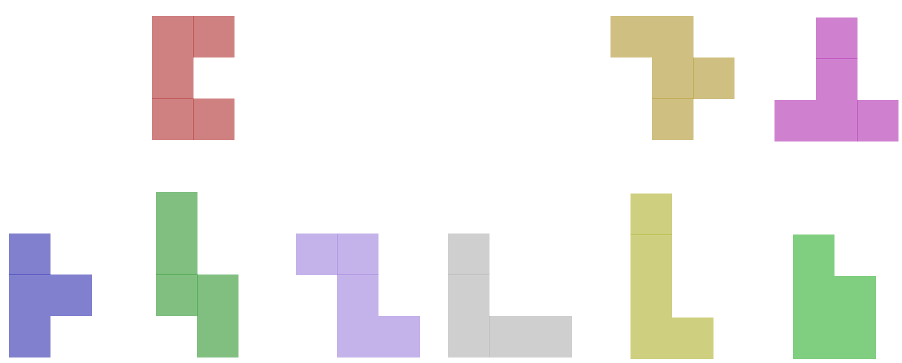


Flipping or rotating a tetromino does not make it a different tetromino. For instance, the four tetrominoes shown below are all considered to be the same tetromino.

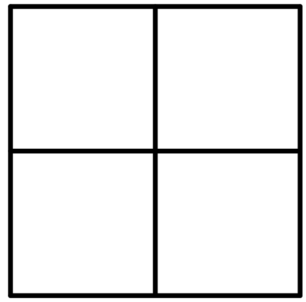


A polynomino made up of 5 squares is called a pentomino. How many different pentominoes are there?

Solution to Question 1. There are $\boxed{12}$ pentominoes, pictured below.



Problem 3. How many total squares are there in a 100 100 grid? How many total squares are there in a $n \quad n$ grid? For example, there are 5 squares in the 2 2 grid shown below.



Solution to Question 3. The number of squares in an $n \quad n$ grid is the sum

$$1^2 + 2^2 + \ldots + (n \quad 1)^2 + n^2 = \boxed{\frac{n(n+1)(2n+1)}{6}}.$$

For $n = 100$, this is $\boxed{338350}$.

Problem 4.

(A.) When I sum five numbers in every possible pair combination, I get the values:

$$0, 1, 2, 4, 7, 8, 9, 10, 11, 12.$$

What are the original 5 numbers?

(B.) When I sum a different set of five numbers in every possible group of 3, I get the values:

$$0, 3, 4, 8, 9, 10, 11, 12, 14, 19.$$

What are the original 5 numbers?

(C.) Is it possible to find a set of 5 numbers that when summed in every possible pair combination results in the sums

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10?$$

Is it possible to find a set of 5 numbers that when summed in every possible group of 3 results in those sums? For each situation, find an example or prove it's impossible.

Solution to Question 4.

(A.) The sum of the pairwise sums is 64, and this counts each of the original five numbers four times, so