Stockton University

Mathematical Mayhem 2018

Group Round - Solutions

April 14, 2018

Name: ____

Name: ____

Name: ____

High School: ____

Instructions:

- This round consists of **5** problems worth **16** points each for a total of **80** points.
- Each of the 5 problems is free response.
- Write your complete solution in the space provided including all supporting work.
- No calculators are permitted.
- This round is **75 minutes** long. **Good Luck!**

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Problem #	1	2	3	4	5	Total
Points Earned						

• Group Round •

Problem 1. To start, three coins are placed in the first three of six squares as shown in the upper diagram. A move consists of moving one coin one space to the right. A move can only be performed if the space to the right of the coin that is moving is not occupied by another coin. How many different sequences of second Y but not in the first positione.

String	Allowed First Z Positions
Y YYXXX	1
Y YXYXX	1
Y YXXYX	1
Y X YYXX	2
Y X YXYX	2

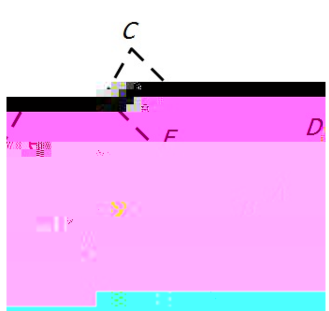
There are 1 + 1 + 1 + 2 + 2 = 7 positions for the first Z. We now place the second Z, and consider the seven length 7 strings made by placing the first Z as shown in the previous table. This time the * will represent possible positions of the second Z, which must be after the first Z but before the last Y.

String	Allowed First Z Positions
YZ Y YXXX	2
YZ Y X YXX	3
YZ Y X X YX	4
YZ X Y YXX	3
YXZ Y YXX	2
YZ X Y X YX	4
YXZ Y X YX	3

Note that there are 2 + 3 + 4 + 3 + 2 + 4 + 3 = 21 length 9 strings that begin with only one Z.

In total we have 5 + 16 + 21 = 42 ways to move the pennies.

Problem 2. The base of a triangular piece of paper ABC is 12 cm long. The paper is folded down over the base, with the crease DE parallel to the base of the paper. The area of the triangle that is below the base after this fold occurs is 16% of the area of the triangle ABC. What is the length of DE? See the figure below which may not be drawn to scale.



Solution to Question 2. Let Z be the bottommost corner of the triangle that is below the fold. Let X and Y be the points at which line segment AB intersects with line segments DZ and EZ respectively. Then the area of $\triangle XYZ$ is 16% of the area of $\triangle ABC$. Notice folding in DE constitutes reflection in DE, and that reflection preserves angle measure. So, $\triangle ACB$ is similar to $\triangle XZY$ since $\angle XZY$ is $\angle ACB$ after reflection and since

$$\angle XYZ = \angle EYB = \angle DEY = \angle CED = \angle CBA$$

by the Alternate Interior Angle Theorem and reflection. Since $\triangle XZY$ is similar to $\triangle ACB$ and its area is $.16 = (.4)^2$ that of $\triangle ACB$, the sides of $\triangle XZY$ are .4 times as long as the sides of $\triangle ACB$.

Draw CZ and let P be the intersection of CZ with $\overleftrightarrow{AB}$ while Q is the intersection of of CZ with $\overleftrightarrow{DE}$. Note that CP is an altitude of $\triangle ACB$. Now,

$$CP = CQ + QP = ZQ + QP = ZP + 2PQ.$$

Since the sides of $\triangle XZY$ are .4 times as long as the sides of $\triangle ACB$, then ZP = 0.4CP65l4